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Fostering Literacy in and via Mathematics

Abstract

Teaching and learning mathematics is often not perceived as being related to literacy, neither by students nor by instructors. However, a lack of certain aspects of literacy might actually severely hinder students' learning of mathematics. This will be exemplified by the analysis of two quite common observations in courses using a specific type of teaching strategy. This teaching strategy is helpful in uncovering literacy-related student difficulties in mathematics as well as providing means to overcome them. It also indicates how to evaluate aspects of students' literacy in a mathematical context during exams.

Keywords

Literacy in mathematics, students' conceptual difficulties, reading strategies, operational definitions, just in time teaching

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Literacy in und durch Mathematik

Zusammenfassung

Lernen und Lehren von Mathematik werden häufig nicht in einem Zusammenhang mit Literacy gesehen. Allerdings stehen bestimmte charakteristische Schwierigkeiten von Studierenden beim Studium der Mathematik womöglich in einer Verbindung mit bestimmten Aspekten von Literacy. Dies wird anhand der Analyse zweier typischer Beobachtungen erläutert, die recht regelmäßig in Kursen gemacht werden können, die eine bestimmte Lehrstrategie verwenden. Diese Lehrstrategie erlaubt nicht nur charakteristische studentische Schwierigkeiten mit mathematischen Inhalten zu identifizieren, sondern begünstigt auch deren Überwindung und ermöglicht bei konsequenter Implementierung das Prüfen von Aspekten von Literacy.

Schlüsselwörter

Literacy in der Mathematik, konzeptuelle studentische Schwierigkeiten, Lesestrategie, operationale Definitionen, Just-in-Time-Teaching

1 Introduction

Studying mathematics literacy and its development are of dual significance since mathematics can be perceived as making use of two languages: formal language and natural language. Literacy aspects tied to the formal language might be specific to mathematics and to other disciplines in engineering and the sciences. The remaining literacy aspects, however, are shared with all academic disciplines. Mathematics also uses written texts to communicate ideas and meaning and does so certainly not to a lesser extent than any other discipline. This paper considers these latter, general aspects of literacy which, more often than not, do not receive much emphasis in teaching mathematics as compared to literacy aspects related to the formal language of mathematics. In general, instructors of mathematics do not deny the importance of these general aspects of literacy. However, for various reasons (RIEGLER, 2014) they often request that literacy should be taken care of elsewhere. In this paper, I wish to argue that it is actually beneficial for learning mathematics if general aspects of literacy are integrated into math courses. In fact, I will argue that there are reasons to do so.

Experiences and findings reported here are derived from various math courses for computer science students at a German university which I have taught over the course of a decade. Section 2 describes the course settings and teaching methodology. It also exemplifies how aspects of literacy are seamlessly integrated into existing courses. The subsequent section describes specific and recurring challenges which relate to the development of literacy and learning mathematics and argues that these are actually interrelated.

The intended readership of this contribution is instructors in mathematics and related topics as well as persons who are in charge of fostering students' literacy in dedicated courses such as writing classes. For the first group I wish to provide evidence and arguments that call for an integration of aspects of literacy into regular courses. I also exemplify how such integration can be accomplished. For the second group I want to provide some concrete cases where a lack of certain aspects of literacy hinders student learning, here in the context of mathematics.

2 Just in Time Teaching and reading assignments

Just in Time Teaching (SIMKINS & MAIER, 2010) is a teaching scenario focusing on helping students to overcome difficulties with subject matter and on helping instructors to identify such difficulties. There is growing research evidence that basically in any discipline there are characteristic "bottlenecks" (PACE & MIDDENDORF, 2004) which students need to pass in their learning process in order to gain expertise. Experts, however, are typically not aware of these bottlenecks. There is an ongoing strand of research which targets at uncovering such bottlenecks. Just in Time Teaching (JiTT) has proven to be helpful with respect to this in many disciplines, including mathematics.

JiTT is related to inverted classroom scenarios in that the information transmission part of teaching is outsourced. While inverted classrooms tend to outsource to video recorded teaching materials (TALBERT, 2014; MCGIVNEY-BURELLE & XUE, 2013), in JiTT students typically are asked to study the subject matter of the upcoming class session using written material such as textbooks. Prior to class students respond to specific reading assignments (to be discussed in detail below) and to answer a number of quizzes. Students' answers to these activities are collected via a course management system. Instructors use these data to analyze what aspects of content matter students are struggling with and to design the upcoming class "just in time" to overcome these barriers. Quite typically more than 60% of students regularly engage in these activities, in particular when using suitably targeted reading assignments and quizzes (HEINER, BANET & WIEMAN, 2014).

Typical reading assignments tell students which sections of the textbook to study and ask them to write up which aspects of the reading they found difficult and want to be covered in class (HENDERSON & ROSENTHAL, 2006). Answers to such open ended reading assignments often uncover and provide deep insights into students' bottlenecks.

Reading assignments obviously help connecting teaching subject matter to literacy. It is important to realize, however, that this connection exists even without them. Traditional teaching also builds on this connection. By copying definitions and theorems from the blackboard students write their own copy of a textbook which they are supposed to study after class. Reading assignments simply make the connection to literacy quite explicit. They do so to the extent that they not only uncover students' difficulties related to subject matter but also their difficulties with literacy in the context of the discipline. In the following section I will discuss a number of such literacy-related characteristic difficulties.

3 Characteristic student difficulties related to literacy in mathematics

3.1 Reading strategies

Mathematical texts typically cannot be read sequentially. This is independent of the type of language used, be it formal or natural language. In the case of formal language, in particular mathematical expressions, this is quite obvious: It is hard to parse an expression like

$$\frac{n!}{k!\,(n-k)!}+\frac{n!}{(k-1)!\,\big(n-(k-1)\big)!}$$

sequentially. Verbalized forms of this such as "a sum of fractions of factorials …" do not follow the left to right sequence of symbols in this expression. Related difficulties of students with reading mathematical expressions are well documented (BURTON, 1988).

In order to parse and, hence, to understand mathematical expressions like the one above it is helpful, if not necessary, to grasp the expression first as a whole and then to zoom into the details. Often the same applies to mathematical prose, in particular when it comes to reading lines of argumentations such as proofs or derivations. It is typically in such situations that students complain in their answers to reading assignments that they could not follow the text. I have found it useful to take this as an opportunity to address this in the upcoming class meeting by eliciting students' reading strategies, contrasting them with my own and by that modeling students' reading strategies.

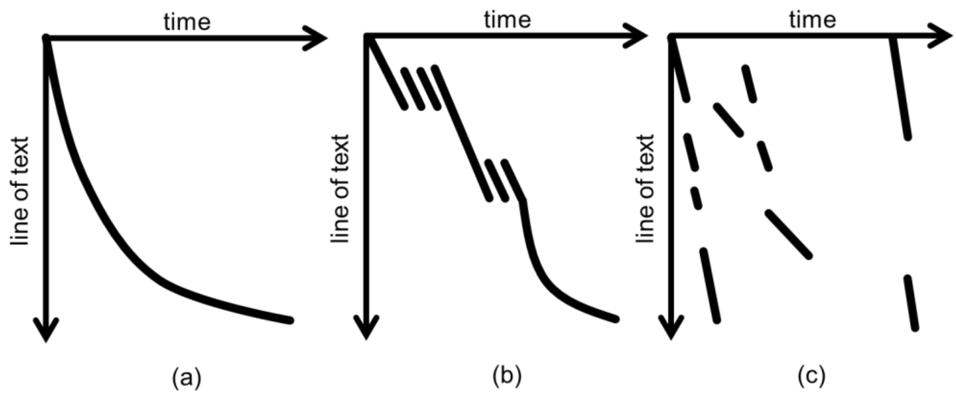

Figure 1: Typical exemplars of students' reading diagrams (a and b), contrasted with the instructor's reading diagram (c).

I do this by asking students to visualize their reading strategy on a piece of paper in a diagram as shown in Fig. 1. These "reading diagrams" qualitatively depict which line of text a student has read at a given instance of time. Regularly, this results in students' reading diagrams most of which are characterized by three features. First, in terms of monotonicity they can be grouped into two classes: reading diagrams that indicate a fully sequential progress such as in Fig. 1a and readings diagrams that indicate that students reread difficult passages several times such as in Fig. 1b. Second, students' reading speed typically decreases, i.e. they need more time to read a certain amount of text the farther they progress in the text. Also students' reading diagrams quite often explicitly indicate that they gave up at a certain point and hence did not read later parts of the assigned reading text. Third, there is striking absence of cases where students read the assigned reading more than once.

These observations indicate that students tend to read mathematical prose sequentially. However, such a reading strategy hardly allows coping with the nested structure which is characteristic for many mathematical texts. This is quite similar to the nested structure of mathematical expressions exemplified above.

After students have visualised their reading strategies I copy characteristic exemplars of students' reading diagrams to the blackboard and ask the students to interpret and to contrast them. After students have analysed and discussed their reading diagrams they are contrasted with a reading diagram showing how I would have read this text, cf. Fig. 1c. Students typically react with disbelief because their instructor's reading diagram appears to be unstructured and haphazard to them. As the classroom discussion evolves they come to realize, however, that this reading strategy focusses on the text as a whole first and recursively zooms into the details as necessary, even omitting parts of the text at times. At this point I demonstrate the corresponding strategy by reading a section of the current reading assignment. While reading I make my thoughts transparent and in particular explain when and why there is a jump in my reading diagram.

The next reading assignment always shows a handful of students who quite enthusiastically report that they started to use a reading strategy characteristic for Fig. 1c, and that they experience an improved understanding with less time commitment. A student poll during the next class meeting regularly reveals that typically about 20% of students took their recent reading assignment as an opportunity to work on their reading strategy.

The teaching strategy just described is certainly in accordance with Bean's advice to explain to students how one's own reading process varies (BEAN, 2011, p. 169). It also enables students to realise that it is not only them as novices who struggle with texts, but that the same applies to experts.

However, I would like to emphasize that simply telling all this to the students would be of little help. Note how the teaching strategy described above takes students' complains about the difficulty of the reading assignment as an opportunity to discuss readings strategies in class. In a way, students' reports of their struggles created teachable moments (SCHWARTZ & BRANSFORD, 1998). It does so by first eliciting students' reading strategies, confronting them with that of an expert, and by that helping them to resolve their reading difficulties. This follows the elicit/confront/resolve process (MCDERMOTT, 1991) which is at the core of many a successful teaching strategy to overcome students' misunderstandings and conceptual difficulties in the sciences.

3.2 Definitions

Definitions play a central role in written mathematical discourse, not only in teaching and learning mathematics. Answers to reading assignments frequently reveal severe difficulties of students with respect to definitions they have encountered in their reading assignments. These can be classified into two categories. First and quite fundamentally, students have difficulties understanding the role of definitions in mathematics (EDWARDS & WARD, 2004) and science in general (ARONS, 1997). Second and as a consequence of the first, they have difficulties applying definitions.

In order to understand students' difficulties with definitions it is helpful to contrast definitions in mathematics with those in encyclopedias. The latter describe the meaning of terms by reporting their usage. They explain terms. The former definitions typically intend to stipulate the usage of a term. They create terms. To provide an example: The mathematical definition of square root does not intend to describe how mathematicians use this term. It stipulates the usage of this term for a certain meaning. In fact, it intends to create the concept square root in the mind of the student. In a way, the definition pretends that neither the related concept nor term had existed before and asks the student to create them in his or her mind for further usage.

Students' difficulties with the nature of definitions manifest themselves in their rather frequent complaints that the assigned text did not explain (i. e. describe) a certain term well enough. Also quite regularly students use their responses to reading assignments to communicate with dismay that they had tried to use other texts or resources (typically videos on the internet) in order to obtain an explanation of a certain meaning.

An important remark is warranted here: These difficulties are neither caused by the textbook nor by JiTT. JiTT only uncovers these difficulties and, hence, provides the opportunity (and obligation) for the instructor to react.

Students need help to understand the stipulatory character of definitions. An activity I have found to be useful for this purpose is to ask students in class whether $\sqrt{x^2}$ equals $\pm x$, $|x|$, or x. A poll on students' answers always results in a dissent which remains even after students discussed this with their neighbors. This helps them to realize that they need to stipulate the meaning of square root in order to avoid what they had just experienced: miscommunication due to the fact that they individually had created different meanings of square root in their minds.

A consequence of the stipulatory nature of many mathematical definitions is their operational character. Definitions list criteria which must jointly be met in order to assign an object to a name. Students need to be provided with frequent opportunities to practice the operational character of definitions. Like others (DUBINSKY, 1997) I have found one way to do so particular useful. Interestingly, it connects the reading aspects of literacy in mathematics with aspects of writing. It also makes use of the duality of natural and formal language in mathematics. I regularly ask students to rewrite textbook definitions, which typically make heavy use of natural rather than formal language, in completely formal form. One way to do that is to write definitions as computer programs. This makes the operational character of definitions very explicit.

Writing definitions as computer programs requires viewing definitions as algorithms which check whether all criteria listed by the definition are satisfied. As a result the computer decides whether a given object is allowed to be named a certain way according to the definition. Note that this also makes the stipulatory nature of definitions very explicit: Writing, i.e. programming definitions, requires students to stipulate the usage of terms with computers.

4 Literacy as an intended learning outcome

The examples given in the previous section indicate that literacy in mathematics cannot be viewed in isolation from subject matter. In fact, one can view literacy as a part of subject matter. This implies that literacy cannot be taught in complete isolation from teaching mathematics as is currently so often the case in university education. Quite to the contrary, at least some elements of literacy need to be integrated into mathematics teaching.

From a Constructive Alignment perspective (BIGGS & TANG, 2007) literacy is an intended learning outcome worthwhile to pursue. Through the usage of JiTT the statement "students are able to acquire mathematical knowledge from texts" had become an intended learning outcome for my math classes. Constructive Alignment also suggests that intended learning outcomes should be aligned with both teaching and testing. JiTT supports alignment of teaching methodology and intended learning outcomes. It gives students opportunities to work on literacy and to receive early feedback on their work. It also encourages meaning-making and allows the instructor to communicate expectations and purpose.

Alignment with testing, however, suggests actually testing students' literacy in exams. This can quite easily be done via text-rich exam items. Such items introduce mathematical concepts new to the student, ask him or her to study the text and subsequently to answer questions related to the newly introduced concept. Specific examples for corresponding exam items can be found in (FRICKE & RIEGLER, 2013).

5 Summary

The teaching experiences discussed in Section 3 strongly indicate that certain frequently observable difficulties of students in studying mathematics are related to literacy issues. Teaching practices such as JiTT or for instance those advocated in (BEAN, 2011) are both helpful and necessary to uncover and possibly overcome such difficulties. Not addressing literacy-related aspects of mathematics with students is likely to result in students missing core concepts of mathematics such as understanding and using definitions.

Hence, studying mathematics requires certain aspects of literacy which in turn need to be fostered to enable students to learn mathematical subject matter. In a way literacy and content are not independent in the learning and teaching of mathematics.

I am grateful to David Pace for carrying out a decoding interview with me. This interview did not only make my own reading process transparent but also helped me to become more sensitive towards students' difficulties related to literacy.

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